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Nonlinear Transient Thermal Stress Analysis of Temperature-Dependent Hollow Cylinders Using a Finite Element Model

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In this paper, the nonlinear transient thermal stress analysis is conducted for temperature-dependent hollow cylinders subjected to a decaying-with-time thermal field. By the finite element method, the highly nonlinear governing equations are solved. The time histories of temperature, displacement, and stress due to the decaying-with-time thermal load are computed. A sensitivity analysis includes the effects of exponent of the decayed heat flux and temperature-dependency of density and material properties is carried out. Numerical results show some interesting characteristics of the thermoelastic behaviors of the hollow cylinders studied. In particular, the effect of temperature-dependency of the material properties on the thermoelastic parameters was demonstrated to be significant.

Keywords: Generalized thermoelasticity; decaying-with-time thermal load; annular cylinder; temperature-dependency; finite element model.

1. Introduction

The numerical value of thermal conductivity varies with temperature, especially for the regions where the change of temperature is large. So, the thermal conductivity, and maybe the heat capacity, should be considered temperature-dependent for most practical engineering problems. The literature contains a number of methods

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developed for the identification of temperature-dependent thermal conductivity.[1–7] In addition to thermal conductivity, material properties like Lamé's elastic coefficients, density, etc. are no longer constant and the temperature dependency of material properties affect the thermo-mechanical behavior of the medium, especially at high temperatures.[8]

One of the main purposes of this article is to investigate the behavior of temperature-dependent infinite cylinders subjected to a thermal field that is decaying with the time. Most of the theoretical investigations reported so far have not taken into account the temperature dependence for the material properties. Therefore, the existing results in general are only adequate for the case with small change of temperature or when the variation of material properties against temperature is insignificant. In fact, the temperature dependency of material properties should be taken into consideration to obtain more dependable solution of the problem of thermoelasticity. Shen and Xiang[9] presented a postbuckling analysis for a cylindrical shell with temperature-dependent material properties. Sun *et al.*[10] developed a new accurate solution methodology to study the torsional bifurcation buckling of functionally graded cylindrical shells in a thermal environment. The effective properties are assumed to be functions of the ambient temperature as well as the thickness coordinate of the shell. The use of temperature-dependent physical properties in estimating the thermoelastic response of cylinders and tubes was assessed by Argeso and Eraslan.[11] In the context of generalized thermoelasticity theory with one relaxation parameter, Mukhopadhyay and Kumar[12] investigated the problem of an infinitely long annular cylinder, whose material properties like modulus of elasticity and thermal conductivity are linearly varying with the temperature.

The theory of classical thermoelasticity was introduced by Biot[13] based upon thermodynamics principles of irreversible processes. This theory predicts a finite speed of propagation for predominantly elastic disturbances but an infinite speed of propagation for predominantly thermal disturbances, which are coupled together. Subsequently, efforts to eliminate this drawback have led to the generalized thermoelasticity theories in which the heat conduction equation is of the hyperbolic type. Lord and Shulman[14] developed a theory of generalized thermoelasticity with one relaxation time by introducing the time derivative of the heat flux vector and a new constant acting as a relaxation time in the classical Fourier's law. The heat conduction equation of this theory is of the wave type, ensuring a finite speed of propagation for both the thermal and elastic waves and thus removes the second drawback of the uncoupled theory. This procedure is called the first generalization to the coupled theory of elasticity for isotropic homogeneous media. The second generalization to the coupled theory of elasticity is what is known as the theory of thermoelasticity with two relaxation times. A generalization of this inequality was proposed by Green and Laws.[15] Green and Lindsay[16] have obtained an explicit version of the constitutive equations. This theory is also called the theory of temperature-rate dependent thermoelasticity, which takes into consideration two relaxation times. Green and Naghdi[17–19] have also proposed three new formulations of thermoelasticity based on inequality of entropy balance.

An infinite cylinder immersed in an infinite medium with thermal surface resistance (contact resistance) at the boundary between the cylinder and surrounding medium is considered as one that is typical in many practical applications. The rotational oscillation of an infinite cylindrical rod immersed in a Newtonian fluid is one of the most important and interesting problems. In addition, the piles used in the foundations of and the long poles used in high-rise buildings are considered as infinitely-long cylinders.

In the present article, the nonlinear transient thermal stress of temperature-dependent infinite cylinders subjected to a decaying with time thermal loading is analyzed. The finite element solution procedure is adopted to solve the highly nonlinear governing equations. Numerical results obtained with the assumptions of temperature-dependent and temperature-independent of the material properties are compared. The variations of temperature, radial displacement, radial stress, and hoop stress with respect to time are plotted. The effect of relaxation time parameter is also studied.

2. Formulation of the Problem

Let us consider an infinite hollow cylinder of inner radius a and outer radius b as shown in Fig. 1. The material properties and density of the cylinder are temperature-dependent. The cylinder is subjected to a thermal field that is decaying with time. The cylindrical polar coordinates (r, θ, z) are used as the coordinates of reference with positive direction along the axis (z) of the cylinder. The present problem is considered to be an axisymmetric plane strain problem. The physical quantities are assumed to be functions of the radial coordinate r and time t only.

The equation of motion in cylindrical coordinates neglecting the body force and the heat conduction equation in the absence of the heat source are given, respectively, by

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}(\sigma_r - \sigma_\theta) = \rho \frac{\partial^2 u}{\partial t^2}, \tag{1}$$

$$K\nabla^2 \bar{T} + \frac{\partial K}{\partial r}\frac{\partial \bar{T}}{\partial r} = \frac{K}{\kappa}\left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)\bar{T} + \gamma T_0 \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)\varepsilon, \tag{2}$$

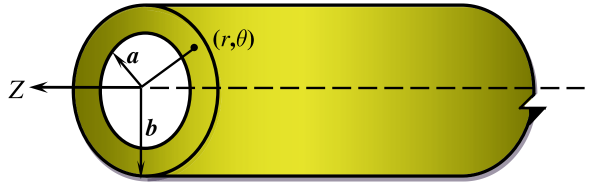

Fig. 1. Geometry of the temperature-dependent infinite cylinder.